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SUPPLEMENTARY NOTE ON MODIFIED-IMPACT-THEORY CALCULATIONS FOR BODIES OF REVOLUTION HAVING MINIMUM DRAG AT HYPERSONIC SPEEDS

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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

WASHINGTON

July 18, 1952

CONFIDENTIAL

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

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SUPPLEMENTARY NOTE ON MODIFIED-IMPACT-THEORY CALCULATIONS FOR BODIES OF REVOLUTION HAVING MINIMUM DRAG AT HYPERSONIC SPEEDS

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SUMMARY

Following the methods of NACA RM A51K27, 1952, the modified impact theory developed therein is employed to obtain improved expressions for calculating the shapes of bodies of revolution having minimum pressure foredrag in hypersonic flight, corresponding to cases where the hypersonic similarity parameter (the ratio of the free-stream Mach number to the slenderness ratio) is appreciably greater than 1. The investigation is carried out for various combinations of the conditions of given body length, base diameter, surface area, and volume. A minimum drag body of given base diameter and surface area is calculated and compared with the cone, the corresponding body obtained with impact theory. It is found that consideration of centrifugal forces in the disturbed flow (with the modified impact theory) yields a shape of increased bluntness in the region of the nose and increased curvature in the region downstream of the nose, which result is in substantial agreement with that already obtained in NACA RM A51K27 for the minimum drag body of given fineness ratio. The calculated pressure drags at hypersonic speeds of the bodies obtained with modified impact theory were only slightly less (of the order of a few percent) than those of the corresponding bodies obtained with the impact theory.

INTRODUCTION

Approximate shapes of nonlifting bodies of revolution having minimum pressure foredrag at high supersonic airspeeds were calculated in reference 1 on the basis of Newtonian impact theory. The investigation was carried out for various combinations of the conditions of given body length, base diameter, surface area, and volume. Comparison between theory and experiment indicated that bodies so calculated do indeed have relatively low drag; however, it was also suggested in reference 1 that centrifugal forces in the disturbed flow about such bodies may

significantly influence their shape, particularly if the value of the hypersonic similarity parameter is appreciably greater than 1 (i.e., at hypersonic flight speeds). The impact theory was therefore modified to account for effects of these forces in hypersonic flight and the corresponding minimum drag body of given length and base diameter was calculated. This body had more bluntness in the region of the nose, and more curvature in the region downstream of the nose than the shape obtained with the impact theory.

Since minimum drag bodies obtained with the modified impact theory would appear to be especially suited for hypersonic flight, it has been undertaken in the present report to extend the calculations of reference 1, using procedures paralleling those presented therein. In particular, the modified theory is employed to develop expressions for calculating minimum drag bodies of given base diameter and body surface area or volume, and given length and volume or surface area.

SYMBOLS

C_D drag coefficient $\left[\frac{D}{q_o\pi(d/2)^2}\right]$

D pressure foredrag

d maximum body diameter

f integrand function

I_D drag parameter $\left(\frac{D}{2\pi q_o}\right)$

l body length

P pressure coefficient $\left(\frac{p-p_o}{q_o}\right)$

p static pressure

q dynamic pressure

S body surface area

V body volume

x,y coordinates of point on meridian curve of body (origin of coordinate system coincides with nose of body, and x axis coincides with axis of symmetry)

δ angle (in meridian plane) between free-stream direction and tangent to body surface

λ Lagrange multiplier

Subscripts

o free-stream conditions

1 values at nose point of minimizing curve

2 values at base point of minimizing curve

$+$ right-hand limiting value of quantity at corner on minimizing curve

$-$ left-hand limiting value of quantity at corner on minimizing curve

PROCEDURE FOR CALCULATING MINIMUM DRAG BODIES

Modified Drag Theory

It was shown in reference 1 that the pressure coefficient obtained by considering only the impact forces may be modified as follows to include centrifugal-force effects in the flow over the surface of a body:

$$P = 2 \sin^2 \delta + \frac{y}{2}\left(1 - \frac{y}{y_2}\right) \frac{d}{dy} \sin^2 \delta \tag{1}$$

where the first term on the right represents the contribution due to impact forces and the second term represents the effect of curvature of the body in the stream direction. A drag parameter may be defined by the relation

$$I_D = \frac{D}{2\pi q_o} = \int_0^l P\, yy'\, dx$$

or, considering equation (1), this expression may be written in the form (see reference 1)

$$I_D = y_2^2 - \int_0^l \left(1 + \frac{3y}{2y_2}\right) \frac{yy'}{1+y'^2}\, dx \tag{2}$$

Variational methods will be used to minimize equation (2), subject to the following conditions:

a. Given base diameter and given surface area
b. Given base diameter and given volume
c. Given length and given volume
d. Given length and given surface area

Minimizing Procedure

Relations for volume and surface area can be written

$$\frac{V}{\pi} = \int_0^l y^2\, dx = \text{const.} \tag{3}$$

and

$$\frac{S}{2\pi} = \frac{y_1^2}{2} + \int_0^l y\sqrt{1+y'^2}\, dx = \text{const.} \tag{4}$$

respectively. The conditions for a fixed volume or surface area may be included in the expression to be minimized by adding a multiple of V/π or $S/2\pi$ to I_D, thus forming a new function, say J_D, to be minimized. Hence,

$$J_D = I_D + \lambda \frac{V}{\pi} = y_2^2 + \int_0^l \left[- \left(1 + \frac{3y}{2y_2}\right) \frac{y'}{1+y'^2} + \lambda y \right] y\, dx \tag{5}$$

for a body with a given volume and

$$J_D = I_D + \lambda \frac{S}{2\pi} = y_2^2 + \lambda \frac{y_1^2}{2} + \int_0^l \left[- \left(1 + \frac{3y}{2y_2}\right) \frac{y'}{1+y'^2} + \lambda \sqrt{1+y'^2} \right] y\, dx \tag{6}$$

for a body with a given surface area, where λ is a constant which will be determined later.

In order to find the forms of the function $y = y(x)$ which will minimize expressions (5) and (6), it is necessary to find solutions to the Euler differential equation (see reference 2)

$$\frac{d}{dx} f_{y'} - f_y = 0 \tag{7}$$

where $f = f(y, y')$ represents the integrand function in equation (5) or (6), and $f_{y'}$ and f_y denote, respectively, $\frac{\partial}{\partial y'} f(y, y')$ and $\frac{\partial}{\partial y} f(y, y')$. It can be readily verified that a first integral to equation (7) is given by

$$y' f_{y'} - f = \text{const.} \tag{8}$$

It follows that a first integral to the Euler equation for the given volume condition is, then,

$$\left[\left(1 + \frac{3y}{2y_2}\right)\frac{2y'^3}{(1+y'^2)^2} - \lambda y\right]y = c \tag{9}$$

and for the given surface-area condition,

$$\left[\left(1 + \frac{3y}{2y_2}\right)\frac{2y'^3}{(1+y'^2)^2} - \frac{\lambda}{\sqrt{1+y'^2}}\right]y = c \tag{10}$$

Solutions to equations (9) and (10), satisfying terminal conditions, represent meridian sections of minimum drag bodies (excluding any finite section of infinite slope at the nose) for the given conditions, and will be called minimizing curves.

Since the ordinate of the minimizing curve at the nose of the body is not specified, a terminal condition must be satisfied (reference 3). For the given volume condition this is

$$0 = f_{y'}\Big|_{x=0} = y_1\left[\left(1 + \frac{3y}{2y_2}\right)\frac{y'^2 - 1}{(1+y'^2)^2}\right]_{y=y_1} \tag{11}$$

and for the given surface-area condition

$$0 = \left(f_{y'} - \lambda \, \frac{d}{dy} \frac{y^2}{2}\right)_{x=o} = y_1 \left[\frac{\lambda y'}{\sqrt{1+y'^2}} + \left(1 + \frac{3y}{2y_2}\right) \frac{y'^2-1}{(1+y'^2)^2} - \lambda \right]_{y=y_1} \tag{12}$$

Similarly, a terminal condition at the base must be satisfied, namely,

$$\left(f_{y'} + \frac{d}{dy} \, y^2\right)_{y=y_2} = 0 \tag{13}$$

when the base diameter is not given, and

$$(y' \, f_{y'} - f)_{x=x_2} = 0 \tag{14}$$

when the body length is not given.

Condition (13) is, for a given volume,

$$y_2 \left[\frac{5}{2} \; \frac{y'^2-1}{(1+y'^2)^2} + 2 \right]_{y=y_2} = 0 \tag{15}$$

and for a given surface area,

$$y_2 \left[\frac{\lambda y'}{\sqrt{1+y'^2}} + \frac{5}{2} \, \frac{y'^2-1}{(1+y'^2)^2} + 2 \right]_{y=y_2} = 0 \tag{16}$$

Equation (14) simply requires that the constant of integration be zero in equation (8), the first integral to Euler's equation.

In addition to the above, two further necessary conditions of the calculus of variations are needed in order to show that each of the various combinations of given conditions determines a unique minimizing curve, namely, the Legendre condition (see reference 2), which requires

$$f_{y'y'} \geq 0 \tag{17}$$

everywhere on the minimizing curve, and the corner condition (see reference 2)

$$f_{y'}\Big|_{-} = f_{y'}\Big|_{+} \tag{18}$$

which must be satisfied if the minimizing curve is to contain a corner. It can be shown that equation (18) cannot be satisfied by the integrand function of expression (5) or (6). It follows, then, that there can be no corners on any of the minimizing curves to be considered, except at end points.

CALCULATION OF MINIMUM DRAG BODIES

Given Base Diameter and Surface Area

The first integral to Euler's equation and the terminal condition at the base (equations (10) and (14), respectively) permit the minimizing curve to be represented parametrically as

$$\left.\begin{aligned} y &= \frac{y_2}{3}\left[\lambda\,\frac{(1+y'^2)^{3/2}}{y'^3} - 2\right] \\ x &= \int_{y_1}^{y}\frac{dy}{y'} = \frac{\lambda y_2}{8}\left[\frac{2(1+y'^2)^{3/2}}{y'^4} + \frac{\sqrt{1+y'^2}}{y'^2} - l\eta\,\frac{1+\sqrt{1+y'^2}}{y'} - c\right] \end{aligned}\right\} \tag{19}$$

The condition at the nose (equation (12)) requires $y_1 = 0$ or $y_1' = \infty$. For all values of base diameter (and surface area) the former requirement gives the drag parameter (see equation (2)) a smaller value. Thus, it is necessary that $y_1 = 0$ and equation (19) yields

$$\lambda = \frac{2y_1'^3}{(1+y_1'^2)^{3/2}} = \frac{5y_2'^3}{(1+y_2'^2)^{3/2}} \tag{20}$$

Using equations (19) and (20), the surface-area condition (4) yields

$$\lambda = 121.6\,(y_2^2/S)^3$$

From equation (20) it is seen that the range of λ is 0 to 2. The corresponding ranges for the length l and surface area S are ∞ to 0.32 y_2 and ∞ to 3.93 y_2^2, respectively.

The minimizing curve for a body of diameter 2 and surface area 31.57 is shown in figure 1. The meridian section of a cone (the corresponding body obtained with the impact theory) of the same base diameter and surface area is also shown in the figure. It is seen that the consideration of centrifugal forces results in a body of greater bluntness in the region of the nose and greater curvature in the meridian section downstream of the nose. Calculation of the drag parameters for these bodies, using equation (2), indicates that the cone will indeed have the higher drag at hypersonic speeds, although not by more than a percent or two. This result and the results of corresponding calculations for the given fineness-ratio bodies treated in reference 1 indicate that consideration of centrifugal forces principally influences the shape and not the drag of minimum drag bodies.

Given Base Diameter and Volume

The first integral to Euler's equation and the terminal condition at the base (equations (9) and (14), respectively) permit the minimizing curve to be represented parametrically by the relations

$$\left.\begin{aligned} y &= \frac{2y_2}{\dfrac{\lambda y_2(1+y'^2)^2}{y'^3} - 3} \\ x &= \int_{y_1}^{y} \frac{dy}{y'} \end{aligned}\right\} \qquad (21)$$

where

$$\lambda y_2 = 5\, y_2'^3/(1 + y_2'^2)^2$$

The nose condition (equation (11)) requires either that $y_1' = 1$, or $y_1' = \infty$, or $y_1 = 0$. If $y_1' = 1$, the resulting l/d ratio would be limited to values less than one-half, since dy/dy' is positive. If $y_1' = \infty$, the Legendre condition would be violated. Thus it is concluded that, at least for l/d ratios greater than one-half, $y_1 = 0$. It follows in this case from equation (21) that $y_1' = 0$. The Legendre condition, namely,

$$f_{y'y'} \geq 0$$

limits the range of y_2' to $0 \leq y_2' \leq \sqrt{3}$. The corresponding range of λ is from 0 to $3\sqrt{3}/16$.

Given Length and Volume

Using the first integral to Euler's equation (equation (9)), the following parametric representation is obtained for the minimizing curve:

$$\left.\begin{aligned} y &= \frac{-y'^3 + \sqrt{y'^6 + c\,(1+y'^2)^2\left[\dfrac{3y'^3}{y_2} - \lambda(1+y'^2)^2\right]}}{\dfrac{3y'^3}{y_2} - \lambda\,(1+y'^2)^2} \\ x &= \int_{y_1}^{y} \frac{dy}{y'} \end{aligned}\right\} \qquad (22)$$

The terminal conditions at the nose and base are given by equations (11) and (15), which yield the values $y_1' = 1$ and $y_2' = 0.274$, respectively, (it can be shown that y_1 cannot be 0).[1]

The range of λ is $-\infty$ to 0, corresponding to the range 0 to ∞ for l. For $\lambda = 0$ this becomes the given length and diameter case (see reference 1).

The numerical integration of equations (22) is accomplished by first evaluating equation (9) at $y = y_2$ and $y_2' = 0.274$ and solving for c/y_2 in terms of $y_2\lambda$. Letting $\varphi(y', y_2\lambda)$ represent the resulting function of y' and $y_2\lambda$, equations (22) now give

$$\left.\begin{aligned} y &= y_2\,\varphi(y', y_2\lambda) \\ l &= \int_{y_1}^{y_2} \frac{dy}{y'} = y_2 \int_{y'=1}^{0.274} \frac{d\varphi(y', y_2\lambda)}{y'} = y_2\,\Lambda(y_2\lambda) \end{aligned}\right\}$$

[1]If y_1 were zero, then it would follow that $c = 0$ in equation (9). Equations (9) and (15) would then prescribe the value of λ. Thus the problem would be overdetermined, that is, the above conditions could not be satisfied for a general body length and volume.

and the volume (equation (3)) is given by

$$\frac{V}{\pi} = y_2{}^2 \int_0^l \varphi^2\,(y',\ y_2\lambda)\ dx = y_2{}^3 \int_{y'=1}^{0.274} \varphi^2\,\frac{d\varphi}{y'}$$

$$= y_2{}^3\ \Gamma\ (y_2\lambda)$$

The values of the functions Λ and Γ are obtained by numerical integration for various (estimated) values of $y_2\lambda$, to enable interpolation for that value of $y_2\lambda$, which makes

$$\frac{\Gamma}{\Lambda^3} = \frac{V/\pi}{l^3}$$

The set $(y_2\lambda,\ \Lambda,\ \Gamma)$ so determined satisfies the given volume and length requirements and yields the base ordinate value

$$y_2 = l/\Lambda$$

Expressions (22) now determine y parametrically as a function of y' and, by numerical integration, x as a function of y'.

Given Length and Surface Area

The first integral to the Euler equation (equation (10)) enables the minimizing curve to be represented parametrically by

$$\left.\begin{aligned} y &= \frac{\left[\lambda(1+y'^2)^{3/2}-2y'^3\right] y_2 \pm \sqrt{\left[\lambda(1+y'^2)^{3/2}-2y'^3\right]^2 y_2{}^2 + 12\,c\,y'^3(1+y'^2)^2 y_2}}{6y'^3} \\ x &= \int_0^y \frac{dy}{y'} \end{aligned}\right\}\quad(23)$$

The terminal conditions at the nose and base are given by equations (12) and (16), respectively. As in the previously discussed case, the problem is overdetermined if $y_1 = 0$. Hence, the body in this case must have a finite blunt nose.

The range of λ is given by

$$-0.64 < \lambda < 2 \left(1 + \frac{3y_1}{2y_2}\right)$$

As λ approaches -0.64, the body length approaches zero.

The procedure used to integrate equations (23) is similar to that used to integrate equations (22) above.

CONCLUDING REMARKS

The modified impact theory of NACA RM A51K27, 1951, was employed to determine improved expressions for calculating the shapes of bodies of revolution having minimum pressure foredrag at hypersonic speeds. Various combinations of the conditions of given body length, base diameter, surface area, and volume were treated. A minimum drag body was calculated for the case of given base diameter and surface area, and was found to be blunter in the region of the nose and to have more curvature in the region downstream of the nose than the corresponding shape, a cone, obtained with impact theory. Centrifugal force effects considered by the modified impact theory were found to influence principally the shape and not the drag of minimum drag bodies.

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REFERENCES

1. Eggers, A. J., Jr., Dennis, David H., and Resnikoff, Meyer M.: Bodies of Revolution for Minimum Drag at High Supersonic Airspeeds. NACA RM A51K27, 1952.

2. Bolza, Oskar: Lectures on the Calculus of Variations. G. E. Stechert and Co. (New York), 1946, pp. 22, 27, 38, 40, and 47.

3. Courant, R., and Hilbert, D.: Methoden der Mathematische Physik. J. Springer (Berlin), vol. 1, 1929, pp. 179-180.

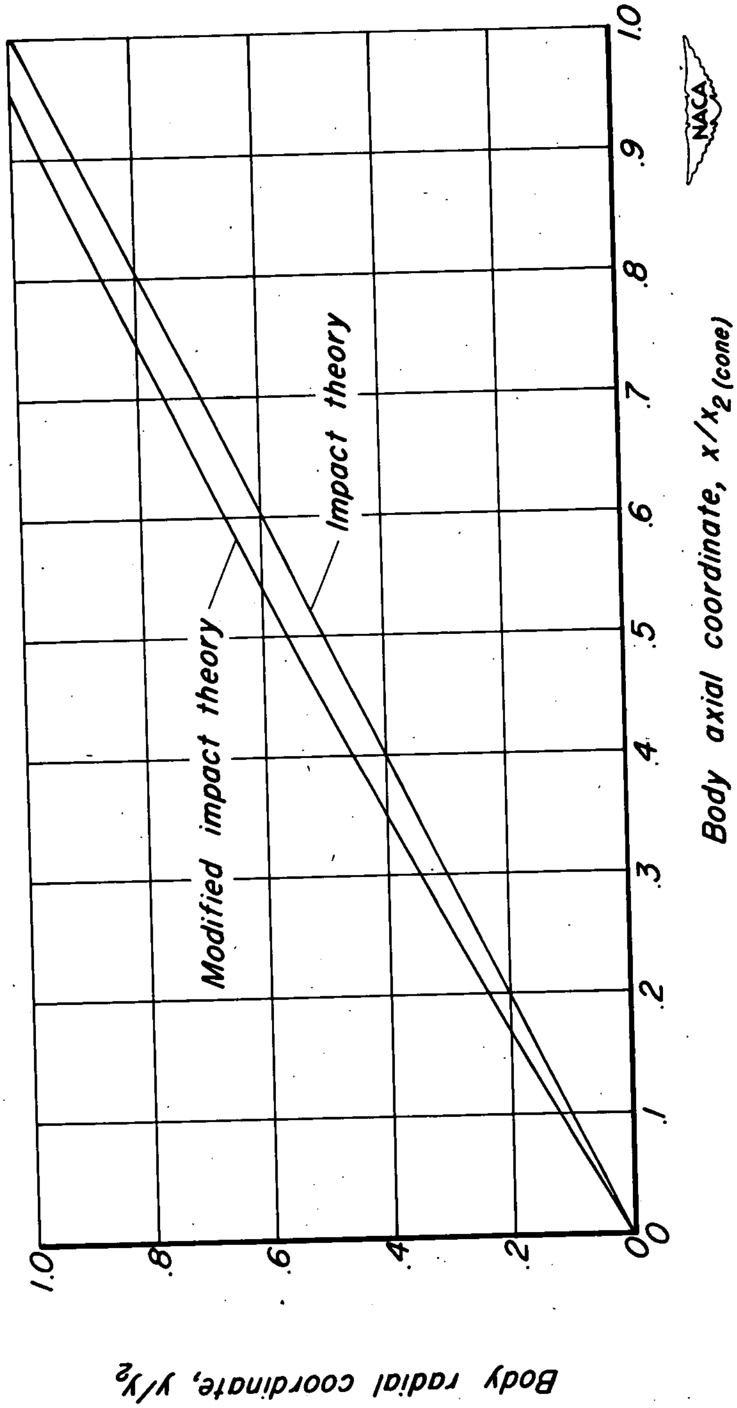

Figure 1.-Comparison of shapes of minimum drag bodies calculated by impact theory and by theory which includes centrifugal force effects ($d = 2$, $S = 31.57$).

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